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STOCHASTIC PRODUCTION SCHEDULING WITH WEIBULL RATE OF PRODUCTION FOR DETERIORATING ITEMS

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Abstract:

Scheduling production process is a prime concern for optimal utilization of resources. Much work has been reported in production scheduling with the assumptions that the production is having finite rate and deterministic. But in many production processes, in particular for deteriorating items the production rate is not deterministic due to various random factors such as availability of raw material, transportation, power supply, manpower and maintenance. The production is random and follows a probability distribution. In addition to this for deteriorating items the deterioration is random. For modelling this phenomenon it is needed to consider the stochastic production scheduling problem with variable rates of production as well as deterioration. This paper addresses this problem by characterizing the production and the life time of commodity with two parameters and three parameters of Weibull distributions respectively. Using the differential calculus, the instantaneous state of inventory is derived the expected total cost function is derived for power pattern demand. By minimizing the expected total cost per unit time, the production start up time, shutdown time and optimal production quantity are derived for without shortages. This model is extended to the case of shortages, where shortages allow and fully backlogged. The sensitivity of the model with respective to variation in input parameters and costs is studied in both the models. The optimal production schedules are sensitive to the changes in the parameters and costs. It is also observed that the

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deterioration parameters have significant influence on optimal production, uptime and downtime. It is also observed that the model with shortages is having less expected total cost per unit time than that of the model without shortages.

Key words: Stochastic production scheduling; Weibull distribution; EPQ model; Sensitivity, analysis.

1 Introduction:

In Chemical and Cement industries the product under production is having finite life time. This life time and the production process are random. But in traditional inventory systems dealing with the production processes assumed that the replenishment rate is finite and fixed. (Deb and Chaudhuri(1986), Bhumia and Maiti(1997)). Deviating from this deterministic production, Noble and Vander Heeden(2000) have considered stochastic production system with two discrete rates of production, Perumal and Arivarignan (2002) studied the production system with two rate of production for different times. Sen and Chakrabarhty (2007) studied an order level inventory model with alternative rates of replenishment. Sridevi et.al(2010) and K.Srinivasa Rao et.al (2011) have studied the stochastic production systems for deteriorating items with Weibull rate of production. However they assumed that the rate of deterioration is constant.

Lakshman Rao et.al(2015 and 2016) have developed inventory models for deteriorating items with weibull rate of replenishment. They assumed that the life time of the commodity follows a generalized pareto distribution. The generalized pareto distribution is having a drawback of decreasing rate of deterioration and do not serve well, when the commodity is having increasing or constant rates of deterioration.

In many practical situations arising at chemical, sea foods and pharmaceutical production processes the rate of deterioration may be increasing or decreasing or constant. Hence, to have an appropriate analysis of the production systems it is needed to consider EPQ models with random production and deterioration.

Very little work has been reported in literature regarding the production scheduling models with the weibull rate of production and deterioration. In this paper we develop and analyze the stochastic production system with the assumptions that production is random and follows a two parameters weibull distribution. It is further assumed that the life time of commodity is also random and follows a three parameter weibull distribution. The weibull distribution is capable of portraying increasing/decreasing/constant rates of production and determination. It also includes exponential distribution as a particular case. Here we consider that the demand is a function of time and follows a power pattern demand. The power pattern demand includes increasing, decreasing or constant rates of demand for different values of indexing parameter.

Using the differential equations the instantaneous state of inventory is derived. The expected loss of deterioration, the expected total cost per unit time are derived. By minimizing the total cot function and using the Hessian matrix, the optimal production up time and down time are derived when shortages are allowed and fully backlogged, through numerical illustrations the solution procedures are demonstrated. The sensitivity analysis with respect to costs and parameters is also performed. This model is extended to the cases of without shortages.

2 Notations and assumptions:

2.1 Notations

The following notations are used for developing the model.

A: ordering cost

C: per unit production cost of the items

T: length of the cycle

Q: Total quantity of items produced in one cycle

K: total profit per unit time in the system

n: pattern index

r: total demand during the cycle

h: inventory holding cost per unit time

π: shortage cost per unit time

t_1: time point at which production stops (production downtime)

t_2: time point at which shortage begins

t_3: time point at which production resumes (production uptime)

Q_1: maximum inventory level

Q_2: maximum shortage level

$\lambda(t)$: demand rate at any time 't'

K(t): production rate at any time 't'

2.2 ASSUMPTIONS:

For developing the Stochastic Production Scheduling model, we consider the following assumptions:

i) The production process is random and follows a Weibull distribution having probability density function of the form

$f(x) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}$; $\alpha > 0$, $\beta > 0$, $t > \gamma$ (1) Then the instantaneous production rate of production is

$$R(t) = \frac{f(t)}{1-F(t)} = \alpha\beta t^{\beta-1}; \quad \text{where } \alpha > 0, \beta > 0 \qquad (2)$$

This production rate includes increasing /decreasing /constant rate of production for Specific values of the parameter β.

ii) The life time of the commodity is random and follow a three parameter Weibull distribution having probability density function of the form

$$g(t) = \theta\eta(t-\gamma)^{\eta-1} e^{-\theta(t-\gamma)^{\beta}} \; ; \; t > \gamma \qquad (3)$$

Therefore the instantaneous rate of deterioration is

$$h(t) = \theta\eta(t-\gamma)^{\eta-1} \qquad (4)$$

Deterioration starts only after the time period γ. This deterioration rate includes increasing/decreasing/constant rates of deterioration for specific values of the parameter η.

iii) The demand rate $\lambda(t)$ is a power function of time which is of the form $\lambda(t) = \frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}}$, 'n' is the indexing parameter, T is the cycle length and 'r' is the total demand, this includes increasing/deceasing/constant rates of demand.

iv) There is no lead time.

v) Cycle length is fixed and known say, T.

vi) There is no repair or replacement of deteriorated item. A deteriorated item is thrown as an absolute.

vii) Money value remains constant throughout the period of production cycle

viii) Shortages are allowed and fully backlogged.

3 STOCHASTIC PRODUCTION SCHEDULULING MODEL WITH SHORTAGES:

Consider a production system with the above assumptions, the stock level in the production system is initially zero at time t=0. The stock level changes with demand and production during the period $(0,\gamma)$. Since deterioration starts after time γ, during the period (γ, t_1) the stock level changes with a mix of production, demand and deterioration. At time t_1, the stock level reaches the maximum and production is stopped. During the period (t_1, t_2) the stock level changes with demand and deterioration, and reaches to zero at time t_2. During the period (t_2,t_3) there is no production, back orders accumulate up to time t_3, and at time t_3 production starts again and fulfils the backlog demand. The backlog demand is completely cleared at time T and stock level reaches to zero. The Schematic diagram representing the instantaneous state of inventory in the production cycle is shown in Fig 1.

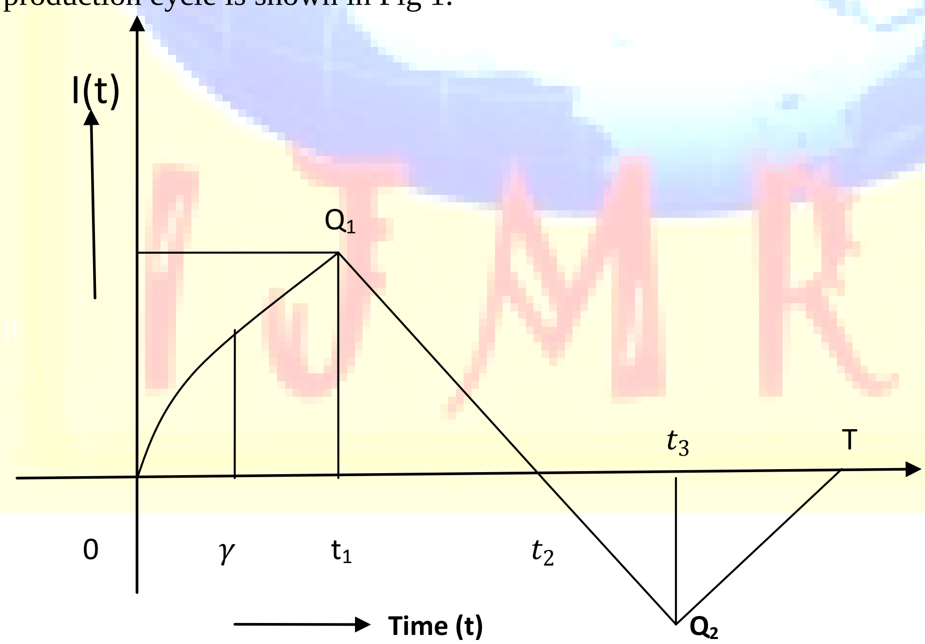


Figure 1 Instantaneous state of inventory in production cycle

Let $I(t)$ denote the inventory level of the system at time `t' $0 \leq t \leq T$. The differential equations describing the instantaneous state of $I(t)$ in the interval (0, T) are:

$$\frac{d}{dt}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; \quad 0 < t \le \gamma \tag{5}$$

$$\frac{d}{dt}I(t) + \theta\eta(t-\gamma)^{\eta-1}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; \quad \gamma < t \le t_1 \tag{6}$$

$$\frac{d}{dt}I(t) + \theta\eta(t-\gamma)^{\eta-1}I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; \quad t_1 < t \le t_2 \tag{7}$$

$$\frac{d}{dt}I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; \quad t_2 < t \le t_3 \tag{8}$$

$$\frac{d}{dt}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; \quad t_3 < t \le T \tag{9}$$

with the initial conditions , $I(0) = 0$, $I(t_2) = 0$, $I(T) = 0$.

Solving the differential equations (5) to (9) using the boundary conditions, the instantaneous state of inventory at any time t, during the interval (0,T) is obtained as

$$I(t) = \alpha t^{\beta} - \frac{rt^{\frac{1}{n}}}{T^{\frac{1}{n}}} \; ; \quad 0 < t \le \gamma \tag{10}$$

$$I(t) = e^{-\theta(t-\gamma)^{\eta}}\left[\left[\int_{\gamma}^{t} \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right] e^{\theta(t-\gamma)^{\eta}} dt + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right]; \quad \gamma < t \le t_1 \tag{11}$$

$$I(t) = e^{-\theta(t-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{t2} t^{\frac{1}{n}-1} e^{\theta(t-\gamma)^{\eta}} dt\right]; \quad t_1 < t \le t_2 \tag{12}$$

$$I(t) = -\frac{r}{T^{\frac{1}{n}}}\left[t^{\frac{1}{n}} - t_2^{\frac{1}{n}}\right] \; ; \quad t_2 < t \le t_3 \tag{13}$$

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} - t^{\frac{1}{n}}\right] - \alpha(T^{\beta} - t^{\beta}) \; ; \quad t_3 < t \le T \tag{14}$$

The expected stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t K(t)dt - \int_0^t \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} dt - I(t) \; ; \quad 0 < t \le t_2 \tag{15}$$

This implies

$$L(t)=\begin{cases}\left[\alpha t^{\beta}-\frac{r\,t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]-e^{-\theta(t-\gamma)^{\eta}}\left[\left[\int_{\gamma}^{t}\left[\alpha\beta t^{\beta-1}-\frac{r t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{\theta(t-\gamma)^{\eta}}\right]dt+\left[\alpha\gamma^{\beta}-\frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right]; & \gamma<t\leq t_1\\ \left[\alpha t_1^{\beta}-\frac{r\,t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]-e^{-\theta(t-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\left[\int_{t}^{t_2}t^{\frac{1}{n}-1}e^{\theta(t-\gamma)^{\eta}}\,dt\right]\right]; & t_1<t\leq t_2\end{cases}$$

(16)

The expected stock loss due to deterioration in the cycle of length ‘T’ is

$$L(T)=\alpha t_1^{\beta}-r\left(\left(\frac{t_2}{T}\right)^{\frac{1}{n}}\right) \quad (17)$$

The expected total production in the cycle of length T is

$$Q=\int_0^{t_1}k(t)dt+\int_{t3}^{T}k(t)dt \quad (18)$$

$$=\alpha\left[t_1^{\beta}+T^{\beta}-t_3^{\beta}\right] \quad (19)$$

when t = t_3, the equations (13) and (14) becomes

$$I(t_3)=\frac{r}{T^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}}-t_3^{\frac{1}{n}}\right] \quad (20)$$

$$I(t_3)=\frac{r}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}}-t_3^{\frac{1}{n}}\right]-\alpha\left(T^{\beta}-{t_3}^{\beta}\right) \quad (21)$$

On equating equations (20) and (21) and on simplification, we get t_2 in terms of t_3 as,

$$t_2=T\left[1-\frac{\alpha}{r}\left[T^{\beta}-t_3^{\beta}\right]\right]^{n}=y\ \text{(say)} \quad (22)$$

Let K (t_1, t_2, t_3) be the expected total cost per unit. Since the total cost is the sum of the set up cost per unit time, purchasing cost per unit time, holding cost per unit time and shortage cost per unit time, thus K(t_1,t_2,t_3) becomes

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left[\int_0^{\gamma}I(t)dt+\int_{\gamma}^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt\right]+\frac{\pi}{T}\left[-\int_{t_2}^{t_3}I(t)dt-\int_{t_3}^{T}I(t)dt\right].$$

(23)

Substituting the values of $I(t)$ and Q given in equations (10) to (14) and (19), in equation from (23), one can obtain K (t_1, t_2, t_3) as,

$$K(t_1,t_2,t_3) = \frac{A}{T} + \frac{C}{T}\ \alpha\left(t_1^{\beta} + T^{\beta} - t_3^{\beta}\right) + \frac{h}{T}\left[\int_0^{\gamma}\left[\alpha t^{\beta} - \frac{r t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]dt\right]$$

$$+\frac{h}{T}\left[\int_{\gamma}^{t_1}\left[e^{-\theta(t-\gamma)^{\eta}}\left[\int_{\gamma}^{t}\left[\alpha\beta t^{\beta-1} - \frac{r t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{\theta(t-\gamma)^{\eta}}dt\right] + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right]dt\right]$$

$$+\left[\int_{t_1}^{t_2} e^{-\theta(t-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{t_2} t^{\frac{1}{n}-1}\, e^{\theta(t-\gamma)^{\eta}}dt\right]dt\right]$$

$$+\frac{\pi}{T}\left[-\int_{t_2}^{t_3}\left[\frac{-r}{T^{\frac{1}{n}}}\left[t^{\frac{1}{n}} - t_2^{\frac{1}{n}}\right]\right]dt + \int_{t_3}^{T}\left[\frac{-r}{nT^{\frac{1}{n}}}\left[T^{\frac{1}{n}} - t^{\frac{1}{n}}\right] - \alpha\left(T^{\beta} - t^{\beta}\right)\right]dt\right] \qquad (24)$$

On integrating and simplifying the equation (24), we obtain K t_1 ,t_2 ,t_3) as,

$$K(t_1,t_2,t_3) = \frac{A}{T} + \frac{C}{T}\ \alpha\ \left(t_1^{\beta} + T^{\beta} - t_3^{\beta}\right) + \frac{h}{T}\left[\left[\frac{\alpha\gamma^{\beta+1}}{(\beta+1)}\right] - \left[\frac{rn\gamma^{\frac{1}{n}+1}}{\left((n+1)T^{\frac{1}{n}}\right)}\right]\right]$$

$$+\left[\int_{\gamma}^{t_1} e^{-\theta(t-\gamma)^{\eta}}\left[\int_{\gamma}^{t}\left[\alpha\beta t^{\beta-1} - \frac{r t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{\theta(t-\gamma)^{\eta}}\right]dt + \left[\alpha\gamma^{\beta} - \frac{r}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}\right]dt\right]$$

$$+\int_{t_1}^{t^2} e^{-\theta(t-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{t^2} t^{\frac{1}{n}-1}e^{\theta(t-\gamma)^{\eta}}dt\right]dt$$

$$+\frac{\pi}{T}\left[\left[\alpha\left[\frac{\beta}{\beta+1}T^{\beta+1} - t_3T^{\beta} + \frac{t_3^{\beta+1}}{\beta+1}\right]\right.\right.$$

$$\left.\left.+r\left[\left(\frac{t^2}{T}\right)^{\frac{1}{n}}\left(\frac{t^2}{n+1} - t_3\right) + \left(t_3 - \frac{T}{n+1}\right)\right]\right]\right] \qquad (25)$$

Substituting the value of t_2 given in equation (22) in equation (25), we obtain the excepted total cost function as,

$$K(t_1,t_3) = \frac{A}{T} + \frac{C}{T}\ \alpha\ \left(t_1^{\beta} + T^{\beta} - t_3^{\beta}\right) + \frac{h}{T}\left[\left[\frac{\alpha\gamma^{\beta+1}}{(\beta+1)}\right] - \left[\frac{rn\gamma^{\frac{1}{n}+1}}{\left((n+1)T^{\frac{1}{n}}\right)}\right]\right]$$

$$+\left[\int_{\gamma}^{t_1} e^{-\theta(t-\gamma)^{\eta}}\left[\int_{\gamma}^{t}\left[\alpha\beta t^{\beta-1}-\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{\theta(t-\gamma)^{\eta}}\right]dt+\left[\alpha\gamma^{\beta}-\frac{r}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}\right]dt\right]$$

$$+\int_{t_1}^{y} e^{-\theta(t-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{y} t^{\frac{1}{n}-1}e^{\theta(t-\gamma)^{\eta}}dt\right]dt$$

$$+\frac{\pi}{T}\left[\frac{\alpha\beta}{(\beta+1)}\left(T^{\beta+1}-t_3^{\beta+1}\right)+r\left[\frac{T}{n+1}\left(1-\frac{\alpha}{\gamma}\left(T^{\beta}-t_3{}^{\beta}\right)^{n+1}-1\right)\right]\right] \quad (26)$$

4 OPTIMAL SCHEDULING POLICIES OF THE MODEL:

In this section, we obtain the optimal policies of the system under study. To find the optimal values of t_1 and t_3, we obtain the first order partial derivative of K (t_1,t_3) given in equation (26) with respect to t_1 and t_3 and equate them to zero. The condition for minimization of K (t_1, t_3) is

$$D=\begin{vmatrix}\frac{\partial^2 K(t_1,t_3)}{\partial t_1^2} & \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3}\\ \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} & \frac{\partial^2 K(t_1,t_3)}{\partial t_3^2}\end{vmatrix}>0 \quad (27)$$

where, D is the determinant of Hessian matrix.

Differentiating K(t_1,t_3) given in equation(26) with respect to 't_1'and equating to zero, we get

$$\frac{C}{T}\left[\alpha\beta t_1^{\beta-1}\right]+\frac{h}{T}\left[e^{-\theta(t_1-\gamma)^{\eta}}\left[\int_{\gamma}^{t_1}\left(\alpha\beta z^{\beta-1}-\frac{rz^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)e^{\theta(z-\gamma)^{\eta}}dz\right]\right.$$
$$\left.+e^{-\theta(t-\gamma)^{\eta}}\left[\alpha\gamma^{\beta}-\frac{r}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}\right]-e^{-\theta(t_1-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t_1}^{y} z^{\frac{1}{n}-1}e^{\theta(z-\gamma)^{\eta}}dz\right]\right]=0 \quad (28)$$

Differentiating K(t_1,t_3) given in equation (26) with respect to 't_3'and equating to zero, we get

$$\frac{C}{T}\left[-\alpha\beta t_3^{\beta-1}\right]+\frac{\pi}{T}\left[-\alpha\beta t_3{}^{\beta}+rT\left(1-\frac{\alpha}{r}\left(T^{\beta}-t_3{}^{\beta}\right)\right)^{\eta}\beta t_3^{\beta-1}\right]=0 \quad (29)$$

Solving the equations (28) and (29) simultaneously, we obtain the optimal time at which the production should be stopped i.e., t_1^* of $t_1$and optimal time t_3^* of t_3, the optimal time at which the production is restarted. By substituting the optimal values t_1^* and t_3^* in equation (19) we get the optimal value of Q as,

$$Q^* = \alpha(t_1^{*\beta} + T^{\beta} - t_3^{*\beta}) \quad (30)$$

5 NUMERICAL ILLUSTRATION

In this section, we discuss the solution procedure of the model through a numerical illustration by obtaining the production uptime, production downtime, optimal ordering quantity and the expected total cost of the system. Here, it is assumed that the commodity is of deteriorating nature and shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model the production parameter 'α' is considered to vary between 105 to108, the values of the other parameters and costs associated with the system are:

A = 3000, 3100, 3200, 3300; C =11, 12, 13; T =12 months; η = 0.50, 0.55, 0.60, 0.65;
β = 0.50, 0.55, 0.60, 0.65; π = 0.5, 0.55, 0.60; h = 0.50,0.51,0.52,0.53;
n =2, 3, 4, 5; r = 50,51,52,53; γ = 0.5,0.55,0.60,0.65; θ = 0.20,0.25,0.30,0.35.

Substituting these values, optimal ordering quantity Q^*, production uptime t_3^*, optimal value of production downtime t_1^* and expected total cost K^* are computed and presented in Table 1

As the ordering cost 'A' increases from 3000 to 3300, the optimal values of production downtime t_1^* increases from 1.940 to 2.017, the optimal production uptime t_3^* increases from 7.994 to 8.610 increases, the optimal ordering quantity Q^* decreases from 213.097 to 204.760, and the expected optimal total cost per unit time K^* increases from 474.166 to 489.649. When the cost per unit 'C' increases from 11 to 13, there is a decrease in optimal ordering quantity Q^* and it decreases from 215.477 to 210.479, the optimal value of production downtime t_1^* increases from 1.894 to 1.999, the optimal production uptime t_3^* increases from 7.774 to 8.256 and the expected total cost per unit time K^* increases from 459.053 to 488.602. If the production parameter 'α' increases from 105 to 108, there is an increase in optimal ordering quantity Q^* from 213.097 to 216.267, the optimal value of production downtime t_1^* increases from 1.940 to 1.964, the optimal production uptime t_3^* increases from 7.994 to 8.197, and the expected total cost per unit time K^* increases from 474.166 to 477.188. When 'β' increases from 0.50 to 0.65 there is an increase in optimal ordering quantity Q^* from 213.097 to 258.340, the optimal value of production downtime t_1^* decreases from 1.940 to1.606, the optimal production uptime t_3^*

increases from 7.994 to 8.210 and the expected total cost per unit time K^* increases from 474.166 to 521.801.The deterioration parameter 'γ' increases from 0.50 to 0.65, there is an increase in optimal ordering quantity Q^* from 213.097 to 213.207, the optimal value of production downtime t_1^* increases from 1.940 to 2.039, the optimal production uptime t_3^* increases from 7.994 to 8.188, and the expected total cost per unit time K^* increases from

A	C	T	α	β	π	h	n	r	γ	θ	η	t_1^*	t_3^*	Q*	K*
3000	**12**	**12**	**105**	**.5**	**.5**	**0.5**	**2**	**50**	**.5**	**.2**	**0.5**	**1.940**	**7.994**	**213.097**	**474.166**
3100												1.970	8.205	210.358	479.327
3200												2.017	8.450	207.616	484.488
3300												2.017	8.610	204.760	489.649
	11											1.894	7.774	215.477	459.053
	12											1.940	7.994	213.097	474.166
	13											1.999	8.256	210.479	488.602
			106									1.947	8.060	214.153	475.174
			107									1.955	8.128	215.210	476.181
			108									1.964	8.197	216.267	477.188
				.55								1.805	8.061	226.258	487.927
				.60								1.707	8.151	241.282	503.704
				.65								1.606	8.210	258.340	521.801
					.50							1.940	7.994	213.097	474.166
					.55							1.863	7.833	213.200	475.236
					.60							1.797	7.693	213.260	476.324
						.51						1.953	8.020	213.111	474.232
						.52						1.967	8.048	213.125	474.297
						.53						1.982	8.076	213.140	474.362
							3					1.864	7.843	213.044	473.413
							4					1.814	7.740	213.018	472.987
							5					1.781	7.673	213.002	472.705
								51				1.942	7.999	213.092	474.171
								52				1.944	8.003	213.088	474.176
								53				1.946	8.007	213.084	474.181
									.55			1.970	8.054	213.134	474.307
									.60			2.003	8.118	213.171	474.441
									.65			2.039	8.188	213.207	474.568
										.25		1.891	7.901	212.978	474.054
										.30		1.851	7.825	212.869	473.947
										.35		1.818	7.762	212.769	473.846
											.55	1.927	7.970	213.059	474.169
											.60	1.914	7.945	213.021	474.172
											.65	1.900	7.920	212.982	474.174

474.166 to 474.568. As the shortage cost per unit time 'π' increases from 0.50 to 0.60, there is an increase in optimal ordering quantity Q^* from 213.097 to 213.260, the optimal value of production downtime t_1^* decreases from 1.940 to 1.797, the optimal production uptime t_3^*decreases from 7.994 to 7.693, and the expected total cost per unit time K^* increases from 474.166 to 476.324. As the indexing parameter 'n' is increasing from 2 to 5, there is a decrease in optimal ordering quantity Q^*, it decreases from 213.097 to 213.002, as the optimal value of production downtime t_1^* decreases from 1.940 to 1.781, the optimal production uptime t_3* decreases from 7.994 to 7.673 and the expected total cost per

TABLE 1

OPTIMAL VALUES OF t_1*, t_3*, Q*, K^* FOR DIFFERENT VALUES OF PARAMETERS WHEN SHORTAGES ARE ALLOWED

unit time K^* decreases from 474.166 to 472.705. As the demand parameter 'r' increases from 50 to 53, the optimal ordering quantity Q^* decreases from 213.097 to 213.084, the optimal value of production downtime t_1^* increases from 1.940 to 1.946, the optimal production uptime t_3^* increases from 7.994 to 8.007, and expected total cost per unit time K^* increases from 474.166 to 474.181. As the inventory holding cost per unit time 'h' increases from 0.50 to 0.53, there is an increase in the optimal ordering quantity Q^* from 213.097 to 213.140, the optimal value of production downtime t_1^* increases from 1.940 to 1.982, the optimal production uptime t_3^* increases from 7.994 to 8.076, and the expected total cost per unit time K^* increases from 474.166 to 474.362.

As the deteriorating parameter 'θ' increases from 0.20 to 0.35, there is a decrease in optimal ordering quantity Q^* from 213.097 to 212.769, the optimal value of production downtime t_1^*decreases from 1.940 to1.818, the optimal production uptime t_3^* decreases from 7.994 to 7.762, and the expected total cost per unit time K* decreases from 474.166 to 473.846. When 'η' increases from 0.50 to 0.65, there is a decrease in optimal ordering quantity Q^*, from 213.097 to 212.982, the optimal value of production downtime t_1^* decreases from 1.940 to 1.900, the optimal production uptime t_3^* decreases from 7.994 to 7.920 and the expected total cost per unit time K^* increases from 474.166 to 474.174.

6 SENSTIVITY ANALYSIS OF THE MODEL

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies by varying each parameter (-15%,-10%,-5%,0%,5%,10%,15%) at a time for the model under study. The results are presented in Table 2.

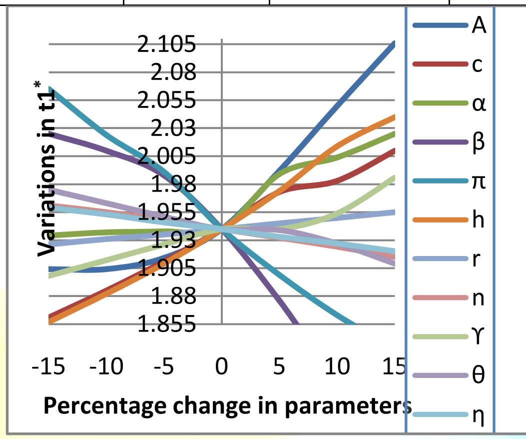
It is observed that costs are having significant influence on the optimal ordering quantity and production schedules. As the ordering cost 'A' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^* and the expected total cost per unit time K^* decrease but the optimal ordering quantity Q^* increases. If 'A' increases, the optimal value of production downtime t_1^*, the optimal production uptime t_3* and the expected total cost per unit time K^* increase but the optimal ordering quantity Q^* decreases. As the cost per unit 'C' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^* and the expected total cost per unit time K^* decrease but the optimal ordering quantity Q^* increases. If 'C' increases, the optimal value of production downtime t_1^*, the optimal production uptime t_3* and the expected total cost per unit time K^* increase but the optimal ordering quantity Q^*decreases. As the production parameter 'α' decreases, the optimal value of production downtime t_1^*, the optimal ordering quantity Q^*, the expected total cost per unit time K^* and the optimal production uptime t_3^* decrease. If 'α' increases, the optimal production downtime t_1^*, the optimal ordering quantity Q^*, the expected total cost per unit time K^*, the optimal production uptime t_3^* increase. As 'β' decreases, the optimal value of production downtime t_1^* increases but the optimal production uptime t_3^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^* decrease. If 'β' increases, the optimal value of production downtime t_1^*decreases but the optimal production uptime t_3^* the optimal ordering quantity Q^* and the expected total cost per unit time K^* increase. As the indexing parameter 'n' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the optimal ordering quantity Q^* and the expected total cost per unit time K* increase. If 'n' increases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the expected total cost per unit time K^* and the optimal ordering quantity Q^* decrease. As the demand parameter 'r' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the expected total cost per unit time K* decrease but the optimal ordering quantity Q^* increases.

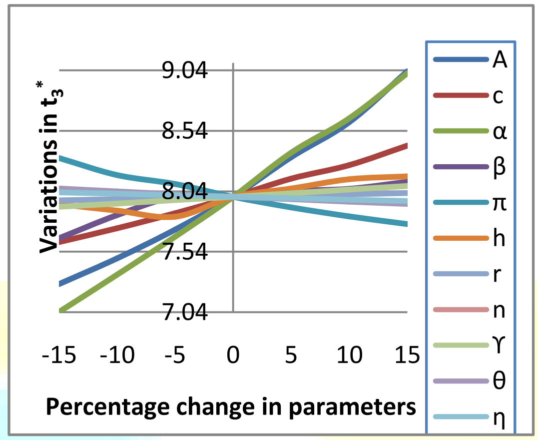
Table 2

SENSITIVITY ANALYSIS OF THE MODEL –WHEN SHORTAGES ARE ALLOWED

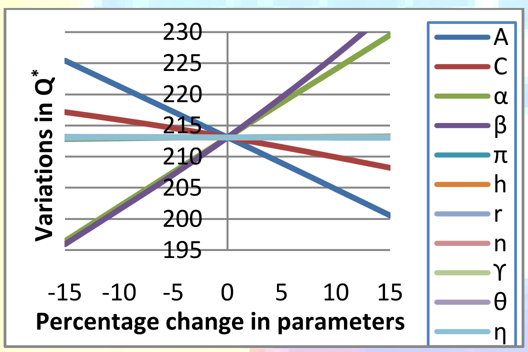
Variation Parameters	Optimal policies	Change in parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
A	t_1*	1.904	1.904	1.914	1.940	1.992	2.017	2.106
	t_3*	7.274	7.486	7.723	7.994	8.323	8.610	9.033
	Q*	225.421	221.313	217.205	213.097	208.989	204.760	200.537
	K*	450.943	458.684	466.425	474.166	481.908	489.649	497.390
C	t_1*	1.861	1.885	1.911	1.940	1.973	1.983	2.017
	t_3*	7.620	7.733	7.858	7.994	8.145	8.257	8.418
	Q*	217.118	215.907	214.557	213.097	211.549	209.879	208.200
	K*	446.440	455.945	465.183	474.166	482.905	491.701	499.701
α	t_1*	1.934	1.937	1.938	1.940	1.989	2.004	2.070
	t_3*	7.042	7.349	7.664	7.994	8.360	8.643	9.015
	Q*	196.450	201.999	207.548	213.097	218.645	224.059	229.494
	K*	458.304	463.592	468.879	474.166	479.453	484.740	490.027
β	t_1*	2.066	2.010	1.988	1.940	1.876	1.805	1.760
	t_3*	7.453	7.844	7.994	7.994	8.040	8.061	8.118
	Q*	195.941	201.464	207.112	213.097	219.478	226.258	233.541
	K*	456.896	462.238	467.987	474.166	480.800	487.927	495.538
π	t_1*	2.091	2.024	1.991	1.940	1.899	1.863	1.833
	t_3*	8.313	8.174	8.100	7.994	7.907	7.833	7.768
	Q*	21025	212.916	213.037	213.097	213.151	213.200	213.244
	K*	472.574	473.103	473.634	474.166	474.701	475.236	475.773
h	t_1*	1.857	1.882	1.909	1.940	1.974	2.014	2.024
	t_3*	7.830	7.880	7.934	7.994	8.062	8.139	8.163
	Q*	212.985	213.023	213.060	213.097	213.132	213.138	213.168
	K*	473.841	474.004	474.079	474.166	474.330	474.493	474.657
r	t_1*	1.926	1.931	1.935	1.940	1.945	1.950	1.955
	t_3*	7.964	7.974	7.984	7.994	8.005	8.015	8.026
	Q*	213.129	213.118	213.107	213.097	213.086	213.075	213.064
	K*	474.130	474.142	474.142	474.166	474.179	474.191	474.203
n	t_1*	1.916	1.955	1.948	1.940	1.932	1.924	1.915
	t_3*	8.035	8.024	8.010	7.994	7.978	7.962	7.945
	Q*	213.123	213.113	213.104	213.097	213.089	213.083	213.076
	K*	474.541	474.403	474.279	474.166	474.064	473.969	473.882
γ	t_1*	1.898	1.912	1.926	1.940	1.955	1.970	1.986
	t_3*	7.912	7.939	7.966	7.994	8.024	8.054	8.085
	Q*	213.039	213.078	213.078	213.097	213.115	213.134	213.153
	K*	473.940	474.093	474.093	474.166	474.166	474.307	474.375
θ	t_1*	1.975	1.963	1.951	1.940	1.929	1.919	1.909
	t_3*	8.062	8.038	8.016	7.994	7.974	7.955	7.936
	Q*	213.172	213.147	213.121	213.097	213.072	213.048	213.024
	K*	474.237	474.213	474.190	474.166	474.143	474.121	474.098
η	t_1*	1.959	1.953	1.946	1.940	1.933	1.927	1.920
	t_3*	8.031	8.019	8.007	7.994	7.982	7.970	7.958

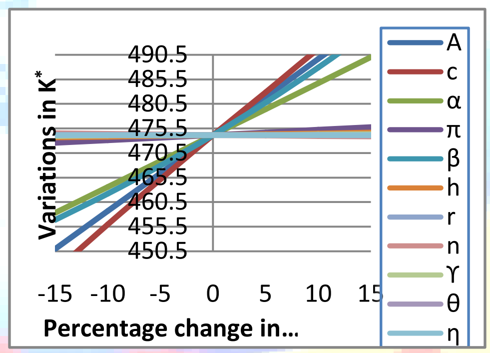
	Q*	213.151	213.133	213.115	213.097	213.078	213.059	213.040
	K*	474.162	474.163	474.165	474.166	474.168	474.169	474.171





(a) **(b)**





(b) **(d)**

Fig 2: Relationship between parameters and optimal values with shortage

If 'r' increases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the expected total cost per unit time K^* increase but optimal ordering quantity Q^* decreases. When the shortage cost per unit 'π' decreases, the optimal value of t_1^* and the optimal production uptime t_3^* increase but the optimal ordering quantity Q^* and the expected total cost per unit time K^*decrease. If 'π' increases, the optimal value of production downtime t_1^* and the optimal production uptime t_3^* decrease but the optimal ordering quantity $Q^*$and the expected total cost per unit time K^* increase.

As the holding cost per unit 'h' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the expected total cost per unit time K^* and the optimal ordering quantity Q^* decrease. If the holding cost per unit 'h' increases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the expected total cost per unit time K^*, and the optimal ordering quantity Q^* increase. As the deterioration parameter 'γ' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the expected total cost per unit time K^* and the optimal ordering quantity Q^* decrease. If 'γ' increases, the optimal value of production downtime t_1^*, optimal production uptime t_3^*, the expected total cost per unit time K^* and the optimal ordering quantity Q^* increase. As the production parameter 'θ' decreases the optimal value of t_1^*, the optimal production uptime t_3^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^* increase. If 'θ' increases the optimal value of production downtime t_1^*, the optimal production uptime t_3^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^* decrease. As 'η' decreases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^* and the optimal ordering quantity Q^* increase but the expected total cost per unit time K^* decreases. If 'η' increases, the optimal value of production downtime t_1^*, the optimal production uptime t_3^* and the optimal ordering quantity Q^* decreases but the expected total cost per unit time K^* increases.

7 STOCHASTIC PRODUCTION SHEDULING WITHOUTSHORTAGES:

In some production processes shortages are not allowed, that is, the production starts as and when the inventory level reaches zero. For this sort of situations, the inventory level changes during $(0,\gamma)$ due to demand and production, during (γ, t_1) due to demand, production and deterioration. At time t=T, the level of inventory reaches zero. The schematic diagram representing the instantaneous state of inventory is given in Figure 3

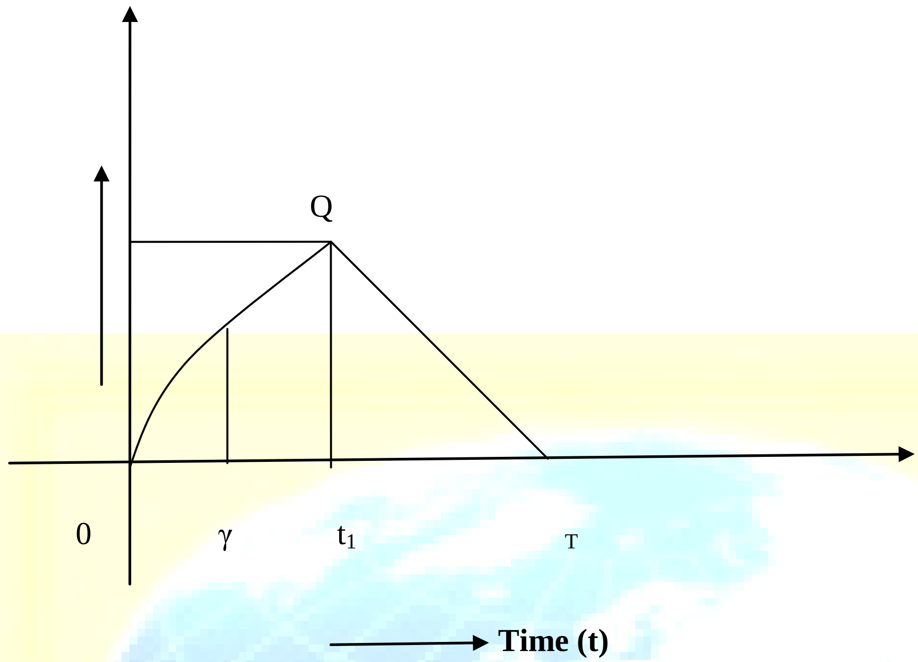


Figure 3 Instantaneous state of inventory in production cycle .

Let $I(t)$ be the inventory level of the system at time ' t', $0 < t \leq T$.

Then the differential equations governing the instantaneous state of $I(t)$ over the cycle of length T are

$$\frac{d}{dt}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \quad ; \quad 0 < t \leq \gamma \qquad (31)$$

$$\frac{d}{dt}I(t) + \theta\eta(t-\gamma)^{\eta-1}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; \quad \gamma < t \leq t_1 \qquad (32)$$

$$\frac{d}{dt}I(t) + \theta\eta(t-\gamma)^{\eta-1}I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; \quad t_1 < t \leq T \qquad (33)$$

with the boundary conditions $I(\mathrm{T}) = 0$, $I(0) = 0$

The instantaneous state of inventory at any given time 't' during the interval (0, T) is

$$I(t) = \alpha t^{\beta} - \frac{rt^{\frac{1}{n}}}{T^{\frac{1}{n}}} \quad ; \quad 0 < t \leq \gamma \qquad (34)$$

$$I(t) = \left[e^{-\theta(t-\gamma)^{\eta}} \left[\int_{\gamma}^{t1} \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right] e^{\theta(t-r)^{\eta}} dt + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}} \right] \right] ;$$

$\gamma < t \leq t_1$ (35)

$$I(t) = e^{-\theta(t-\gamma)^{\eta}} \left[\frac{r}{nT^{\frac{1}{n}}} \int_{t}^{T} t^{\frac{1}{n}-1} e^{\theta(t-\gamma)^{\eta}} dt \right]; \qquad t_1 < t \leq T \qquad (36)$$

The expected stock loss due to deterioration in the interval (0,t) is

$$L(t)=\int_0^t K(t)dt - \int_0^t \lambda(t)dt - I(t); \qquad 0 < t \leq T \qquad (37)$$

This implies

$$L(t) = \begin{cases} \left[\alpha t^{\beta} - \frac{r\, t^{\frac{1}{n}}}{T^{\frac{1}{n}}} \right] - e^{-\theta(t-\gamma)^{\eta}} \left[\left[\int_{\gamma}^{t} \left[\alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right] e^{\theta[t-\gamma]^{\eta}} \right] dt \right] \\ + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{nT^{\frac{1}{n}}} \right]; & \gamma < t \leq t_1 \\ \left[\alpha t_1^{\beta} - \frac{r\, t^{\frac{1}{n}}}{T^{\frac{1}{n}}} \right] - e^{-\theta(t-\gamma)^{\eta}} \left[\frac{r}{nT^{\frac{1}{n}}} \left[\int_{t}^{T} t^{\frac{1}{n}-1} e^{\theta(t-\gamma)^{\eta}} dt \right] \right]; & t_1 < t \leq T \end{cases}$$

(38)

Using equations (34) to (36), we obtain the expected stock loss due to deterioration in the interval (0, T) as the difference between the total quantity produced and the demand met during (0, T) and is given by

$$L(T) = \alpha t_1^{\beta} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} T \qquad (39)$$

This amount of quantity is lost due to deterioration of commodity and is a waste, to obtain the optimal operating policies one must reduce the stock loss due to deterioration.

The production quantity during the cycle time (0, T) is given by

$$Q = \alpha t_1^{\beta} \qquad (40)$$

Let K (t_1) be the expected total cost per unit time. The expected total cost is the sum of the set up cost, cost of units and the inventory holding cost. Therefore the expected total cost is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{\gamma} I(t)dt + \int_{\gamma}^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right] \quad (41)$$

Substituting the values of $I(t)$ and Q given in equations (34) to (36) and (40) in the equation (41) we obtain K (t_1) as

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\alpha\left(t_1^{\beta}\right) + \frac{h}{T}\left[\left[\int_0^{\gamma} \alpha t^{\beta} - \frac{rt^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]dt + \int_{\gamma}^{t_1}\left[e^{-\theta(t-\gamma)^{\eta}}\left[\int_{\gamma}^{t} \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}dt\right]e^{\theta(t-\gamma)^{\eta}}dt + \left(\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right)\right]dt + \left[\int_{t1}^{T}\left[e^{-\theta(t-\gamma)^{\eta}}\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{T} t^{\frac{1}{n}-1}e^{\theta(t-\gamma)^{\eta}}dt\right]dt\right]\right] \quad (42)$$

On integrating and simplifying equation (42), we get

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\alpha\left(t_1^{\beta}\right) + \frac{h}{T}\left[\left(\frac{\alpha\gamma^{\beta+1}}{(\beta+1)}\right) - \left[\frac{nr\gamma^{\frac{1}{n}+1}}{\left((n+1)T^{\frac{1}{n}}\right)}\right]\right] + \int_{\gamma}^{t_1}\left[e^{-\theta(t-\gamma)^{\eta}}\left[\left[\int_{\gamma}^{t} \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]\right]e^{\theta(t-\gamma)^{\eta}}dt + \left(\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right)\right]dt + \int_{t1}^{T}\left[e^{-\theta(t-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t}^{T} t^{\frac{1}{n}-1}e^{\theta(t-\gamma)^{\eta}}dt\right]\right]dt \quad (43)$$

8 OPTIMAL OPERATING POLICIES OF THE SYSTEM

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, we obtain the first order partial derivative of $K(t_1)$ with respect to t_1 and equating to zero. The condition for minimization of $K(t_1)$ is

$$\frac{\partial^2 K(t_1)}{\partial t_1^2} > 0 \qquad (44)$$

Differentiating K (t_1) with respect to 't_1'and equating to zero, we get

$$\frac{C}{T}\left(\alpha\beta t_1^{\beta-1}\right)+\frac{h}{T}\left[e^{-\theta(t1-\gamma)^{\eta}}\left[\left[\int_{\gamma}^{t_1}\alpha\beta z^{\beta-1}-\frac{\gamma z^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{\theta(z-\gamma)^{\eta}}dz\right]+e^{-\theta(t_1-\gamma)^{\eta}}\left[\alpha\gamma^{\beta}-\frac{r}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}\right]\right.$$
$$\left.-e^{-\theta(t_1-\gamma)^{\eta}}\left[\frac{r}{nT^{\frac{1}{n}}}\int_{t_1}^{T}z^{\frac{1}{n}-1}e^{\theta(z-\gamma)^{\eta}}dz\right]\right]=0 \qquad (45)$$

Solving equation (45) we obtain the optimal time at which the production is to be stopped t_1^* of t_1^* and by substituting the optimal values of t_1 in (40) we obtain the optimal value of Q as,

$$Q^* = \alpha t_1^{*\beta} \qquad (46)$$

9 NUMERICAL ILLUSTRATION

In this section, we discuss a numerical illustration of the model. For demonstrating the solution procedure of the model, the production parameter 'α' is considered to vary from 105 to 108, the values of other parameters and costs associated with the model are:

A = 3000,3200,3400,3600; C =10,10.5,11; β = 0.50,0.525,0.550,0.575; n =2,3,4,5; γ = 0.50,0.60,0,70,0.80; r = 50,51,52,53; h = 0.50,0.55,0.60,0.65; T =12months θ = 0.2,0.3,0.4,0.5; η = 0.50,0.55,0.60,0.65;

From the Table 3, it is observed that the deterioration parameters and production parameters have a tremendous influence on the optimal values of the model.

It is observed that costs are having significant influence on the optimal ordering quantity and production schedules. As the ordering cost 'A' is increases from 3000 to 3600, the optimal ordering quantity Q* decreases from 160.962 to 145.485, the optimal value of production downtime t_1^*decreases from 2.350 to 1.920, and the expected total cost per unit time

K^* increases from 389.289 to 425.451. As the cost per unit 'C' increases from 10 to 11, the optimal ordering quantity Q^* decreases from 160.962 to 156.368, the optimal value of production downtime t_1^* decreases from 2.350 to 2.218, and the expected total cost per unit time K^* increases from 389.289 to 398.205. As the production parameter 'α' increases from 105 to 108, the optimal ordering quantity Q^* increases from 160.962 to 161.992, the optimal value of production downtime t_1^* decreases from 2.350 to 2.250, and the expected total cost

TABLE 3

OPTIMAL VALUES OF t_1*, t_3*, Q*, K^* FOR DIFFERENT VALUES OF PARAMETERS WHEN SHORTAGES ARE NOT ALLOWED

A	C	T	α	β	γ	r	h	n	θ	η	t_1^*	Q^*	K^*
3000	**10**	**12**	**105**	**.5**	**.5**	**50**	**.5**	**2**	**.2**	**.5**	**2.350**	**160.962**	**389.289**
3200											2.185	155.201	400.796
3400											2.042	150.042	412.851
3600											1.920	145.485	425.451
	10										2.350	160.962	389.289
	10.5										2.284	158.682	393.858
	11										2.218	156.368	398.205
			106								2.315	161.291	389.544
			107								2.282	161.634	389.812
			108								2.250	161.992	390.093
				.525							2.470	168.780	395.949
				.550							2.616	178.186	403.981
				.575							2.791	189.457	413.618
					.60						2.354	161.088	389.854
					.70						2.357	161.205	390.829
					.80						2.360	161.314	390.783
						51					2.356	161.162	389.986
						52					2.362	161.361	390.683
						53					2.367	161.559	391.383
							.55				2.470	168.780	395.949
							.60				2.616	178.186	403.981
							.65				2.791	189.457	413.618
								3			2.350	160.815	388.571

								4			2.350	160.710	388.034
								5			2.350	160.633	387.633
									.3		2.325	160.094	388.166
									.4		2.304	159.388	387.229
									.5		2.288	158.815	386.446
										.55	2.345	160.785	389.133
										.60	2.340	160.609	388.976
										.65	2.335	160.433	388.820

per unit time K^* increases from 389.289 to 390.093. When 'β' increases from 0.500 to 0.575, the optimal ordering quantity Q^* increases from 160.962 to 189.457, the optimal value of production downtime t_1^* increases from 2.350 to 2.791, and the expected total cost per unit time K^* increases from 389.289 to 413.618. As the indexing parameter 'n' increases from 2 to 5, the optimal ordering quantity Q^* decreases from 160.962 to 160.633, the optimal value of production downtime t_1^* remains constant at 2.350, and the expected total cost per unit time K^* decreases from 389.289 to 387.633. As the demand parameter 'r' increases from 50 to 53, the optimal ordering quantity Q^* increases from 160.962 to 161.559, the optimal value of production downtime t_1^* increases from 2.350 to 2.367, and the expected total cost per unit time K^* increases from 389.289 to 391.383. Inventory holding cost per unit time 'h' increases from 0.50 to 0.65, the optimal ordering quantity Q^* increases from 160.962 to 189.457, the optimal value of production downtime t_1^* increases from 2.350 to 2.791, and the expected total cost per unit time K^* increases from 389.289 to 413.618. As the deteriorating parameter 'γ' increases from 0.50 to 0.80, the optimal ordering quantity Q^* increases from 160.962 to 161.314 the optimal value of production downtime t_1^*, increases at 2.350 to 2.360, and the expected total cost per unit time K^* increases from 389.289 to 390.783. As the deteriorating parameter 'θ' increases from 0.20 to 0.50, the optimal ordering quantity Q^* decreases from 160.962 to 158.815, the optimal value of production downtime t_1^* decreases from 2.350 to 2.288, and the expected total cost per unit time K^* decreases from 389.289 to 386.446. As 'η' increases from 0.50 to 0.65, the optimal ordering quantity Q^* decreases from 160.962 to

160.433, the optimal value of production downtime t_1^* decreases from 2.350 to 2.335, and the expected total cost per unit time K^* decreases from 389.289 to 388.820.

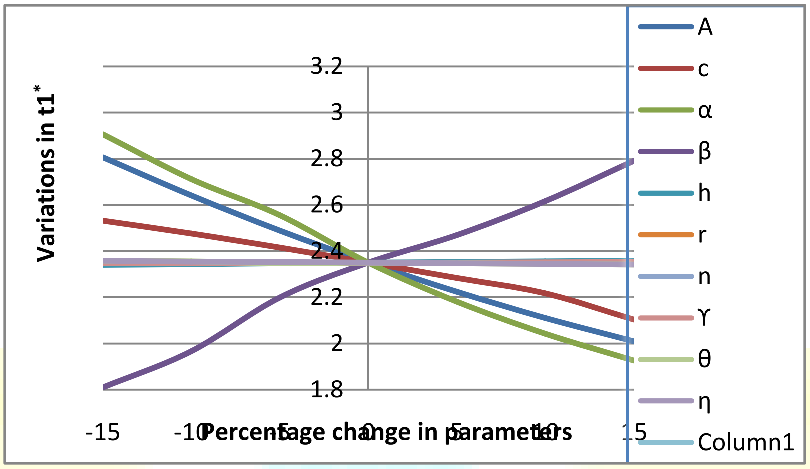
10 SENSITIVITY ANALYSIS OF THE MODEL

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies by varying each parameter (-15%,-10%,-5%,0%,5%,10%,15%) at a time for the model under study. The results are presented in Table 4.

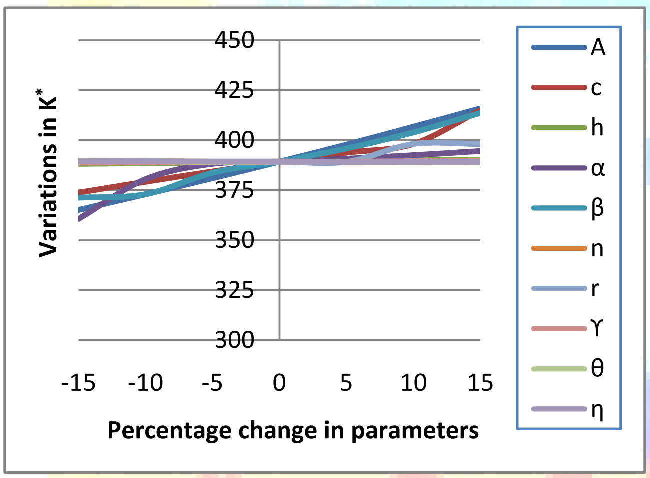
As the ordering cost 'A' decreases the optimal value of production downtime t_1^*, the optimal ordering quantity Q^* increase, and the expected total cost per unit time K^* decreases. If 'A' increases, the optimal value of production downtime t_1*, the optimal ordering quantity Q* decrease, and the expected total cost per unit time K* increases. As the cost per unit 'C' decreases, the optimal value of production downtime t_1*, and the optimal ordering quantity Q* increase, the expected total cost per unit time K^* decreases. If 'C' increases, the optimal value of production downtime t_1*, and the optimal ordering quantity Q* decrease and the expected total cost per unit time K* increases. As the indexing parameter 'n' decreases, the optimal values of production downtime t_1*, the optimal ordering quantity Q* and the total cost per unit time K^* also increases. If 'n' increases, the optimal value t_1^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^* also decrease. As the demand parameter 'r' decreases, the optimal value of production downtime t_1^*, the optimal ordering quantity $Q^*$and the expected total cost per unit time K^*increase. If 'r' increases, the optimal value of production downtime t_1^*, the optimal ordering quantity Q^* and the total expected cost per unit time K^* decrease. As the holding cost per unit 'h' decreases, the optimal value of production downtime t_1^*, the optimal ordering quantity $Q^*$and the expected total cost per unit time K^*decrease. If 'h' increases, the optimal value of production downtime t_1^* and the optimal ordering quantity Q^* and the expected total cost per unit time K^* increase. As the production parameter 'α' decreases, the optimal value of production downtime t_1^*

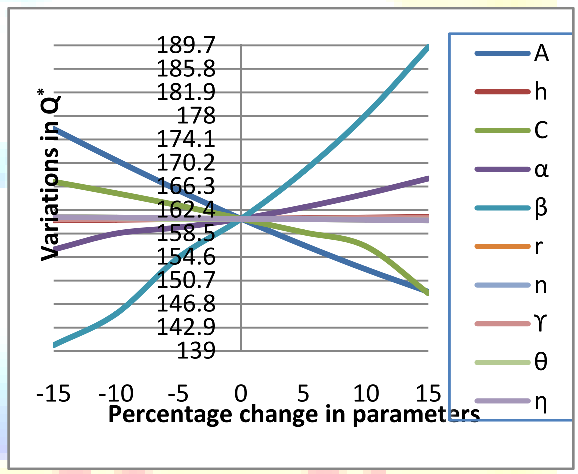
TABLE 4
SENSTIVITY ANALYSIS OF THE MODEL WITHOUT SHORTAGES

Variation Parameters	Optimal Policies	Changes in Variable						
		-15%	-10%	-5%	0%	5%	10%	15%
A	t_1*	2.806	2.641	2.489	2.350	2.224	2.111	2.010
	Q*	175.875	170.638	165.656	160.962	156.585	152.545	148.847
	K*	365.192	372.978	381.001	389.289	397.869	406.754	415.951
C	t_1*	2.532	2.476	2.415	2.350	2.284	2.218	1.604
	Q*	167.071	165.211	163.156	160.962	158.682	156.368	148.568
	K*	373.888	379.334	384.459	389.289	393.858	398.205	400
α	t_1*	2.906	2.812	2.556	2.350	2.182	2.043	1.927
	Q*	155.829	158.469	159.488	160.962	162.845	165.080	167.607
	K*	340.719	387.492	388.182	389.289	390.770	392.569	394.635
β	t_1*	1.967	1.962	2.255	2.350	2.470	2.611	2.791
	Q*	139.964	142.185	154.508	160.962	168.780	178.186	189.457
	K*	371.259	372.994	383.815	389.289	395.949	403.981	413.618
r	t_1*	2.351	2.351	2.350	2.350	2.350	2.349	2.349
	Q*	160.996	160.984	160.973	160.962	160.951	160.940	160.928
	K*	389.440	389.390	389.339	389.289	389.239	389.189	389.139
n	t_1*	2.352	2.351	2.351	2.350	2.350	2.349	2.349
	Q*	161.015	160.997	160.979	160.962	160.945	160.929	160.913
	K*	389.538	389.454	389.371	389.289	389.209	389.131	389.054
h	t_1*	2.341	2.344	2.347	2.350	2.353	2.356	2.359
	Q*	160.660	160.761	160.861	160.962	161.062	161.162	161.262
	K*	388.248	388.595	388.942	389.28	389.637	389.986	390.334
γ	t_1*	2.347	2.348	2.349	2.350	2.351	2.352	2.353
	Q*	160.859	160.894	160.928	160.962	160.995	161.027	161.058
	K*	388.839	388.994	389.144	389.289	389.431	389.569	389.703
θ	t_1*	2.359	2.356	2.347	2.350	2.347	2.345	2.342
	Q*	161.258	161.158	160.867	160.962	160.867	160.774	160.683
	K*	389.669	389.540	389.168	389.289	389.168	389.048	388.930
η	t_1*	2.358	2.355	2.353	2.350	2.347	2.345	2.342
	Q*	161.225	161.138	161.050	160.962	160.874	160.785	160.697
	K*	389.521	389.445	389.367	389.289	389.211	389.133	389.055



(a)





(b) (c)

Fig 4: Relationship between parameters and optimal values without shortages

increases but the optimal ordering quantity Q^* and the expected total cost per unit time K^* decrease. If 'α' increases, the optimal value of production downtime t_1^* decreases but the optimal ordering quantity Q^* and the expected total cost per unit time K^* increase. As the production parameter 'β' decreases, the optimal value of t_1^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^* decrease. If 'β' increases, the optimal value of production downtime t_1^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^* increase. As the deterioration parameter 'γ' decreases, the optimal value of production downtime

t_1^*, the optimal ordering quantity Q^* and the expected total cost per unit time K^*decrease. If 'γ' increases, the optimal value of production downtime t_1^*, the optimal ordering quantity $Q^*$and the expected total cost per unit time K^* increase. As the deterioration parameter 'θ' decreases, the optimal values of production downtime t_1^*, optimal ordering quantity $Q^*$and the expected total cost per unit time K^* increase. If the deterioration parameter 'θ' increases, the optimal values of production downtime t_1^*, optimal ordering quantity Q^* and the expected total cost per unit time K^* decrease. As 'η' decreases, the optimal value of t_1^*, the optimal ordering quantity $Q^*$and the expected total cost per unit time K^* increase. If 'η' increases, the optimal value of production downtime t_1^*, the optimal ordering quantity $Q^*$and the expected total cost per unit time K^*decrease.

11 Conclusions:

This paper addresses the stochastic scheduling of a production system when the production and life time of the commodity are random. The model presented here is of generic in nature including various patterns of production and deteriortion.The instantanenous state of inventory is derived. With sutiable cost consideraions the expected cost per unit time is obtained. By minimizing the total cost function per a unit time, the optimal production uptime and downtime are derived. The sensitivity analysis of the model revealed that the deteriorating distribution parameters, production distribution parameters and demand parameters have significant influence on production scheduling. The operation manager can estimate the production distribution parameters, the deteriorating distribution parameters, using histroical data. It is also observed that the cost can influence the optimal scheduling. This model is extented to the case of without shortages. This model also includes some of the early models as particular cases for specific values of the parameters. It is possible to extented this model with infletion where the money value changes with time, which will be taken up elsewhere. The proposed model is useful for scheduling the production processes in which the product under consideration is deteriorating in nature and the production is subject to random.

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